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***Abstract*—We study millimeter wave based ranging of randomly located terminal nodes (TN) using fixed relay nodes (RN) deployed around a central node (CN). This setting may correspond to a disaster-relief scenario where the rescuers require positioning information in the absence of a global positioning system (GPS). We derive the Bayesian Cramer-Rao lower bound (BCRLB) for the TNs range estimation from the CN as well as from the RNs in this network using a stochastic geometry framework. Contrary to existing studies, we take the effect of link-blockages into account while deriving the BCRLB, and thereby present a more accurate bound on the ranging error. For the special case of no blockages, we formulate a convex problem for obtaining the optimal relay positions. Our results provide the operator a guideline for initial deployment planning, in terms of number and location of RNs to be deployed in order to achieve an accurate ranging.**

I. Introduction

Reliability-constrained wireless applications, especially those pertaining to disaster-relief are becoming more important every day. Such use cases often require cyber-physical systems capable of operating with wireless links of high resilience, for which today's wireless networks are not optimized. One challenging issue in deploying such systems is the accurate positioning of the terminal nodes (TNs) of the network [1]. In contrast to the traditional positioning techniques [2], localization using millimeter wave (mm-wave) signals offers a more attractive solution, precisely due to the higher temporal resolution and high directivity [3]. However, signals at such high frequencies suffer from high transmission losses and high sensitivity to blockages [4]. In case the communication link state changes from line of sight (LOS) to non-LOS (NLOS), severe degradation of localization performance may occur, which may prove detrimental. The aim of this letter is to study the enhancement of the mm-wave ranging performance of the TNs through cooperation. This is facilitated by anchoring some of the TNs and employing them as relay nodes (RNs) to perform the positioning of other randomly located TNs.

Lemic et. al. [5] have shown that mm-wave signals can be used for very accurate positioning of nodes, even with a limited number of anchor nodes. Although extensive literature is available pertaining to RNs for communication reliability enhancement in ad-hoc wireless networks, only a few focus on RNs for localization (e.g., see Dardari et al. [6]). In this regard, Zhang et al. [7] provided a brief survey of cooperative localization in 5G networks, where they discussed the benefit of exploiting the location information from additional localization measurements between TNs. However, the authors only provided a schematic for performance evaluation in terms of localization accuracy and did not provide any quantifiable gain. Coluccia *et al.* [8] have recently investigated RSS-based localization via Bayesian ranging and iterative least squares positioning. They have compared several range-based techniques in terms of their accuracy and computational cost. The evaluation of the statistical localization performance for random positions of TNs has relatively sparse literature. One recent work by O'Lone et al. [9] has statistically characterized the localization performance of a single user placed anywhere throughout a wireless network, where anchors are distributed according to a Poisson point process (PPP).

The contributions of this work can be summarized as follows. We study the ranging accuracy of randomly located TNs, characterized by their Bayesian Cramer-Rao lower bound (BCRLB) of ranging from the perspective of a central node (CN), and the nearest RN. We have accounted for the path-loss and the signal degradation due to physical blockage on the resulting bounds, which are generally ignored in the literature [6]. Leveraging on this, and using tools from stochastic geometry, we mathematically characterize the enhancement in the ranging performance of the TNs by employing some TNs as RNs. Finally, we optimize the RN positions for the case without blockages and study the effect of the number of RNs in the network on the joint ranging accuracy of the TNs. Thus, our framework can be used for initial deployment planning for such a network.

Notations: In this letter, the positions of the nodes are depicted by bold font (e.g., $\boldsymbol{x}$), and the scalar variables are depicted by lowercase and uppercase normal math font (e.g., x and R). Sets are denoted by math calligraphic font (e.g., $\mathcal{S}$). The symbols $||.||$ refer to the norm of the argument (e.g., $||\boldsymbol{x}_n - \boldsymbol{x}_0||$ is the distance between $\boldsymbol{x}_n$ and $\boldsymbol{x}_0$).

II. System Model

We consider the deployment area to be a closed bounded subset $\mathcal{S}$ of the two-dimensional Euclidean plane. For simplicity, we can consider $\mathcal{S}$ to be a diskof radius R. A controlling CN is assumed to be located at the center of $\mathcal{S}$. Furthermore, we assume that there are N_T TNs distributed uniformly in $\mathcal{S}$. Generally, search and rescue operations consist of a fixed and limited number of nodes [10]. Accordingly, we assume N_T to be fixed. Thus, the positions of the TNs: $\{\boldsymbol{x}_i\}$, $i = 1, \dots N_T$, form a binomial point process (BPP).

To increase the reliability of ranging through cooperation, N_R (≥ 2) TNs are employed as RNs, which feedback local ranging measurements to the CN using a reliable channel. Consequently, $N = N_T - N_R$ are the remaining TNs. The RNs are assumed to be located at known positions $\boldsymbol{r}_j \in \mathcal{S}$, $j = 1, \dots N_R$, each at a distance $||\boldsymbol{r}_j|| = p$ (which we optimize in our work) from the CN and separated from each other by an angle of $\frac{2\pi}{N_R}$. The placement of the CN at the center follows the assumption of uniform distribution of the users in the area. In a real deployment, depending on the actual TN positions, the network operator needs to further fine-tune the CN and TN positions for that network. It must also be noted that the presence of a single CN relates to a single point of failure for the network operation which may prove to be detrimental in case of search and rescue operations. It is thus important to note that our work can be easily extended to multiple interconnected CNs located at designated positions of the network. Naturally, the ranging performance would improve with multiple CNs. Although our model allows for the non-isotropic placement of RNs, due to the symmetry of the problem, we focus on a single sector and we define the x-axis as the straight line between the CN and the RN of the sector. Consequently, we simplify the notation and let $(p, 0)$, $0 \leq p \leq R$ be the coordinate of the RN on the x-axis in the studied sector. Accordingly, we use the terms *RN position* and *RN distance* interchangeably.

We assume that the mm-wave communication links are blocked by physical objects locations of which are modeled as atoms of a point process independent of the TN BPP (see e.g., [11] for discussions on blockages and their effects on data-communication). The probability of a TN located at $\boldsymbol{x}_i$ to be in LOS with respect to a reference point $\boldsymbol{x}_0$, is denoted by $p_L(||\boldsymbol{x}_i - \boldsymbol{x}_0||) = \exp(-\beta||\boldsymbol{x}_i - \boldsymbol{x}_0||)$ [11], where β is the blockage density. Since highly resilient signals are required for accurate ranging and mm-wave signals can suffer a high degree of penetration loss due to blockages [4], we assume that ranging over blocked links is unfeasible. Let G_C, P_C and G_R, P_R be the directional antenna gains and the transmit powers of the CN and RN, respectively. Our aim is to study the optimal value of p for a given N_R and N_T from the perspective of ranging accuracy of the TNs. The ranging accuracy is calculated in terms of theoretical bound on the error of estimation (i.e., the BCRLB) of the distance of the TNs from either the RN or the CN.

Before deriving our main results, we would like to state that in this work, we derive the BCRLB by considering the order statistics of the TNs rather than assuming uniform distribution of each TN. The main motivation for using the order statistics of the TN locations is to characterize individual ranging performance of the TNs as a function of their order. This is useful when the operator wants to guarantee ranging performance for individual nodes.

III. BCRLB Formulation

In this section, first we derive the relevant distance distributions of the TNs from the CN and the RN. Let us assume that the indexes of the TNs are arranged in increasing order of their distances from the CN, i.e., $||\boldsymbol{x}_i|| < ||\boldsymbol{x}_k||$ for $i < k$. From the perspective of the CN, we have the following lemma.

Lemma 1. *The pdf of the distances of the n^{th} closest TN (with location $\boldsymbol{x}_n$) to the CN is given by:*

$$f_{\boldsymbol{x}_n}^C(x) = \frac{2}{R} \frac{\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(N+1\right)}{\Gamma\left(n\right)\Gamma\left(N+\frac{3}{2}\right)} \mathcal{B}\left(\frac{x^2}{R^2}; n+\frac{1}{2}, N-n+1\right),$$

where $\Gamma(\cdot)$ and $\mathcal{B}(\cdot)$ are the gamma, and the beta functions, respectively.

Proof. This follows from the void probabilities of BPP [12]. □

Then, for a TN located at a distance $||\boldsymbol{x}_n||$ from the CN, the distribution of its distance from the RN is derived in the following Lemma.

Lemma 2. *For a TN located at a distance $||\boldsymbol{x}_n||$ to the CN at an angle ψ from the x-axis, the probability density function (PDF) of its distance to the RN, conditioned on ψ, is:*

$$f_{\boldsymbol{x}_n}^R(y|\psi) = F_1(y,\psi)\left(f_{\boldsymbol{x}_n}^C\left(p\cos\psi + \sqrt{y^2 - p^2\sin^2\psi}\right) + f_{\boldsymbol{x}_n}^C\left(p\cos\psi - \sqrt{y^2 - p^2\sin^2\psi}\right)\right)$$

where $F_1(y,\psi) = \frac{y}{\sqrt{y^2 - p^2 sin^2 \psi}}$. Here, ψ is uniformly distributed between 0 and $\frac{\pi}{N_R}$.

Proof. For a TN present at a distance x and at an angle ψ from the CN, its squared distance from the RN is: $y^2 = x^2 + p^2 - 2xp\cos(\psi)$. As a result the cumulative density function (CDF) can be calculated as:

$$\begin{aligned} F_Y(y|\psi) &= \mathbb{P}\left(Y \leq y|\psi\right) = \mathbb{P}\left(x^2 + p^2 - 2xp\cos\psi \leq y^2\right) \\ &= \mathbb{P}\left((x - p\cos\psi)^2 \leq y^2 - p^2\sin^2\psi\right) \end{aligned}$$

We now consider two cases[1]. Case I: $x \geq p\cos\psi$, for which: $F_Y(y|\psi) = \mathbb{P}\left(x \leq p\cos\psi + \sqrt{y^2 - p^2\sin^2\psi}\right) = F_X^C(p\cos\psi + \sqrt{y^2 - p^2\sin^2\psi})$. And case II: $x < r\cos\psi$, for which: $F_Y(y|\psi) = \mathbb{P}\left(p\cos\psi - p < \sqrt{y^2 - p^2\sin^2\psi}\right)$ $= \mathbb{P}\left(p > r\cos\psi - \sqrt{y^2 - p^2\sin^2\psi}\right) = 1 - F_X^C(p\cos\psi - \sqrt{y^2 - p^2\sin^2\psi})$. The result then follows from adding the two possibilities and differentiating with respect to y. □

Leveraging on the distance distributions, we calculate the BCRLB of the TNs as given in the following theorem.

Theorem 1. *The BCRLB for the estimation of the distance of the n^{th} TN from the CN or the RN using averaged received signal strength indicator (RSSI) is calculated as:*

$$B_t(\boldsymbol{x}_n) = \left(\mathbb{E}_{d_{n,t}}\left[\sigma_t^{-2}(\boldsymbol{x}_n) + \frac{d}{dx}\log\left(f_{\boldsymbol{x}_n}^t(x)\right)\right]\right)^{-1}, \quad (1)$$

where, $d_{n,t} := ||\boldsymbol{x}_n - \boldsymbol{x}_t||$ and $t = \{C, R\}$ depends on whether the estimating node is the CN or an RN. The distance distribution is obtained as $f_{d_{n,t}}(y,\psi) = f_{\mathbf{x_n}}^t(y|\psi) f_\psi(\psi)$, where $f_\psi(\psi)$ follows uniform distribution of ψ between 0 and $\frac{\pi}{N_R}$. In this expression, $\sigma_t^2(\boldsymbol{x}_n)$ is given by (2), which is the minimum variance of an unbiased estimator of the range of a TN from $\boldsymbol{x}_t$ (a CN or an RN).

Proof. Let the RSSI measurement at the locating node (RN or CN) of a TN be perturbed by a zero mean additive white Gaussian noise (AWGN), $\mathcal{N}$, of variance σ_n^2. The average RSSI thus can be written as:

$$Y = KP_tG_t d_{n,t}^{-\alpha}\exp\left(-\beta d_{n,t}\right) + \mathcal{N}, \quad (3)$$

[1]Note that y is always greater than or equal to $p\sin\psi$ as the later is the perpendicular distance from the TN to the x-axis.

$$\sigma_t^2\left(\boldsymbol{x}_n\right)=\left[\frac{d_{n,t}^{-\alpha-2}\exp\left(-\beta d_{n,t}\right)}{2\sigma_n^2}\left(\beta^2KP_tG_td_{n,t}^2+2\alpha\beta KP_tG_td_{n,t}+\alpha^2KP_tG_t\right)\right]^{-1}; \tag{2}$$

where α is the path-loss exponent and K is the path-loss coefficient.

Let $\mu=Y-KP_tG_td_{n,t}^{-\alpha}\exp\left(-\beta d_{n,t}\right)$ that is distributed as $\mathcal{N}(0,\sigma_n^2)$. The likelihood is thus:

$$\mathcal{L}\left(\mu\right)=\frac{1}{\sqrt{2\pi\sigma_n^2}}\cdot \exp\left(-\frac{\left(y-KP_tG_td_{n,t}^{-\alpha}\exp\left(-\beta d_{n,t}\right)\right)^2}{2\sigma_n^2}\right)$$

As a result, the Fischer information can be derived as:

$$J=\frac{\partial^2\ln\left(\mathcal{L}\left(\mu|x_n\right)\right)}{\partial d_{n,t}^2}=\frac{d_{n,t}^{-\alpha-2}\exp\left(-\beta d_{n,t}\right)}{2\sigma_n^2}\cdot \left[\beta^2KP_tG_td_{n,t}^2+2\alpha\beta KP_tG_td_{n,t}+\alpha^2KP_tG_t\right]. \tag{4}$$

The second term in (1) is the prior information of the distances of the TNs. Inverting the sum of the prior information and J, we obtain the BCRLB. □

Note that given a number of relays being deployed in the network area, the expected value of the BCRLB for the estimation of the angle of arrival (AoA), ψ, given by $\mathbb{E}_\psi\left[\sigma_\psi^2\right]$ is a multiple of σ_d^2 [13], [14]. As a result the ranging performance metric can be easily extended to a localization performance metric by re-writing the objective function as a linear combination of σ_d^2 and $\mathbb{E}_\psi\left[\sigma_\psi^2\right]$.

IV. RN Placement Problem

The BCRLB provides us a convenient objective for the RN placement problem, as it is the expected value of the minimum variance of an unbiased estimator of the TN ranges. However, for reliable applications, merely a constraint on the expected ranging performance of the TNs does not suffice, and performance guarantees on the ranging performance of each TN must be addressed. Accordingly, we formulate the optimal RN placement problem as:

$$\begin{aligned} p^* = \quad & \underset{p}{min} && \sum_{i=1}^{N}\min\{B_C(\boldsymbol{x}_i),B_R(\boldsymbol{x}_n)\} \\ & \text{subject to} && 0\le p\le R \\ \forall i\in\{1,\dots,N\}, & && \mathbb{P}\left(\min\{B_R(\boldsymbol{x}_i),B_C(\boldsymbol{x}_i)\}\le\epsilon\right)\ge\eta \end{aligned} \tag{5}$$

The objective is the sum of the point-wise minimum of the ranging error bounds of the TNs from the CN and the nearest RN, respectively. The constraints on the individual BCRLBs give a performance guarantee of η that the ranging accuracy is at least ϵ for any realization of $\{\boldsymbol{x}_n\}$.

Evidently, the problem (5) is difficult to solve due to several issues. The objective function contains the prior terms, $\mathbb{E}\left[\log\left(f_{\boldsymbol{x}_n}^t(x)\right)\right]$ which are non-convex. Moreover, the point-wise minimum in the final n constraints are also not convex, thereby making the overall problem difficult to tackle. As a result, we present a modified problem that uses the expected distances of the TNs for acquiring a coarse estimate of the RN ranges. For the special case of $\beta=0$, the modified problem becomes a convex optimization problem. For the general case, the modified problem avoids the calculation of the point-wise minimum in the objective function. Let the expected locations of the TNs be given by $\hat{\boldsymbol{x}}_n$. Then, the corresponding distances from the CN can be denoted as i.e, $||\hat{\boldsymbol{x}}_n||=\int_0^R xf_{\boldsymbol{x}_n}^C(x)$.

A. Certainty Equivalent (CE) Formulation

As the BCRLB increases with increasing distance of the CN and the TN (this can be easily seen by differentiating (4) with respect to $d_{n,t}$), we note that there exists an index k such that we have $B_C(\hat{\boldsymbol{x}}_i)<\epsilon$, $\forall i\le k$. Thus, for all TNs that are nearer to the CN than $||\hat{\boldsymbol{x}}_k||$, the BCRLB for the estimation of their ranges from the CN is less than ϵ. In other words, the distances of these TNs are efficiently estimated by the CN. Accordingly, we propose a modified problem that considers the placement of RNs for the TNs whose distances from the CN are larger than $||\hat{\boldsymbol{x}}_k||$. The modified problem given as:

$$\begin{aligned} p^* = \quad & \underset{p}{min} && \sum_{i=k}^{N}B_R(\hat{\boldsymbol{x}}_i) \\ & \text{subject to} && 0\le p\le R \\ & \forall i=k,\dots,N && B_R(\hat{\boldsymbol{x}}_i)\le\epsilon \end{aligned} \tag{6}$$

where, $k=\text{argmax}_n\mathbb{1}\left(B_C(\hat{x}_n)<\epsilon\right)$. Thus, the modified problem (6) essentially calculates the optimal RN placements with respect to the average ranges of the TNs $k,k+1,\dots,N_T$.

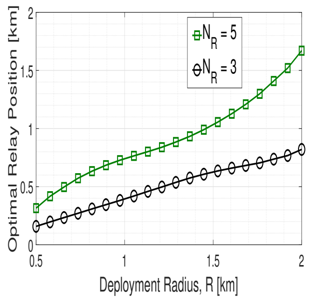
Corollary 1. *For $\beta=0$, each term of the objective function in* (6) *and the last N-k constraints are of the form $2\sigma_n^2\alpha^2KP_tGd_{n,t}^{\alpha+2}$, which is convex with respect to p. Thus, the problem* (6) *is convex for the special case of no blockages.*

The solution of (6) may not be optimal for several realizations of the TN positions, as they are governed by their distance distributions rather than the expected values. However, following the solution of (6), the operator can perform a robust estimate for the RN placement. In other words, the solution to (6) acts as a guideline for an operator for initial planning of such a network. This can be further fine-tuned according to the specific environment and use case.

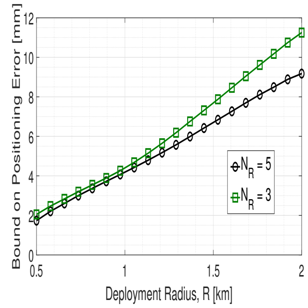
Although a detailed discussion of the stochastic placement of the RNs is out of scope of this letter, in the next section, we present some numerical results that show how the placement of RN following the solution of (6) even for the case without blockages, improves the localization performance of the TNs. Furthermore, we also show that depending on the area of the deployment region, the gain in localization performance by adding more RNs saturates after a certain number of RNs.

V. Numerical Results and Discussion

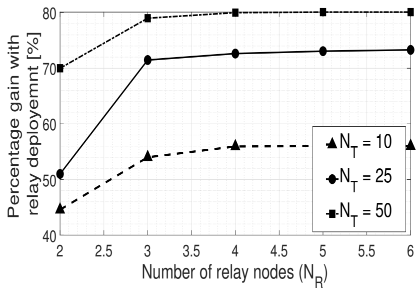
For evaluating the system performance, we have assumed $P_C=30$ dBm, $P_R=20$ dBm, $G_C=20$ dBi, and $G_R=10$ dBi. The path-loss coefficient K is derived from the 3GPP specifications [15]. The path-loss exponent are assumed to be $\alpha=2$ for LOS propagation. We assume that ranging is



(a)



(b)



(c)

Fig. 1. (a) Optimal RN placement for different number of RNs, (b) Bound on the ranging error vs deployment radius, and (c) Percentage gain in localization accuracy with cooperation as a function of the number of RNs.

infeasible for NLOS links. The blockage parameter is assumed to be $\beta = 0.007\ \text{m}^{-1}$ [16]. We assume a carrier frequency of 60 GHz and a bandwidth of 1 GHz. In Fig. 1a we plot the optimal RN placement for $N_R = 3$ and $N_R = 5$. Interestingly, we see that as the number of RNs increase, the optimal placement of the RNs from the center increases. This is because a smaller N_R results in a larger sector. As an analogy, we can recall that the center of mass of a sector of a uniformly dense disk is nearer to the center of the disk if the sector is larger. Similarly, with uniformly distributed TNs, for larger sectors (i.e., with less number of RNs), the RNs should be placed nearer to the center to guarantee good ranging performance of the TNs located farther from the x-axis in the representative sector. In Fig. 1b, we plot the BCRLB of the sum of the mean ranging errors of the TNs with optimally placed relays, with respect to the deployment radius. As the deployment radius increases, the ranging accuracy decreases due to increasing number of TNs located farther from the CN. More interestingly, in case of small deployment radius (e.g., $R = 1$ km), the gain with more number of RNs (5 as compared to 3) is not appreciable.

In Fig. 1c we plot the percentage gain in ranging performance by cooperation as a function of the number of RNs in a 1 km radius. Mathematically, it is represented as:

$$\%Gain = \frac{B^* - B_0}{B_0} \times 100 \tag{7}$$

where B^* is the sum of BCRLBs of ranging errors from the RNs, i.e., the the objective function in (6) for the case when relays are optimally placed, whereas, B_0 is the BCRLB of the ranging errors of the TNs for the case when only CN performs the ranging. First we note that with cooperation, the ranging performance increases significantly (by upto 80%). Furthermore, the additional gain with more than 4 RNs is not appreciable. Thus, as long as four quadrants are covered by the RNs, the limits of gain for localization with cooperation is achieved. In case the operator wants to improve the ranging performance even further, she/he must deploy more effective cooperative algorithms, since deploying more RNs simply saturate the gain. It must be noted that for a larger deployment area, the number of RNs to achieve the gain limit may increase. Thus, a study to account for the cost of using more TNs as RNs, from the deployment perspective is necessary, which we will address in a future work.

VI. Conclusion

We have investigated the placement of relay nodes to augment the ranging accuracy of the terminal nodes in a mm-wave network. Appropriate placement of the relays for local ranging may improve the ranging performance by up to 80 percent as compared to utilizing only the central node. The ranging accuracy gain saturates when using a large number of relays. Our results provide the operator a guideline for initial planning in terms of the number and placement of relays to support given ranging accuracy requirements. In this regard, a parallel study of the cost of relay provisioning is essential, which we plan to carry out as future work.

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